Trajectory of the wave field centroid

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The behavior of a wave-field centroid with spatial dynamics described by the Schrödinger equation is investigated analytically and numerically. The consideration was made for the case of media with inertia of the nonlinear response. The peculiarities of the dynamics of wave-beam self-action in the media with focusing and defocusing nonlinearities were analyzed. Within a paraxial optics approximation, the behavior of the centroid trajectory was studied qualitatively. The numerical calculations of the peculiarities of a wave-beam self-action dynamics under the conditions of nonrectilinear motion of the centroid were carried out. [S1063-651X(97)02805-5]

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INTRODUCTION

The study of the dynamics of self-action for electromagnetic waves and spatially localized pulses is usually restricted to the consideration of axially symmetric field distributions. One major reason for such an approach is connected, in the long run, to the rectilinearity of the trajectory of the wave field centroid. However, only in the case of the nonlinear Schrödinger equation with a local nonlinearity can one provide a strict proof that nonlinearity does not change the linear trajectory of the centroid (the first momentum of the wave-field intensity). Because of this, the basic features of self-action dynamics are determined only by the behavior of the efficient pulse distribution width (second momentum of the intensity) [1]. One cannot make such a general conclusion even for media with a nonlocal dependence of the potential on the field amplitude in the Schrödinger equation and all the more for media with inertia of the nonlinear response. Recent studies [2,3] on the self-action of a spatially limited pulse in plasmas under excitation of a wake field demonstrated the unstable motion of the centroid, called ''hose-modulation instability'' [2]. One can expect that the unstable motion of the centroid is inherent for the dynamics of the self-action of wave beams in any medium with nonlinear relaxation.

This work presents analytical and numerical considerations of the behavior of the intensity center and dynamics of the wave-field structure in the medium, with a nonlinearity that is described by a relaxation equation. Peculiarities of the processes considered have been studied for the cases of both focusing and defocusing nonlinearities. The approach developed to determine the self-consistent trajectory of the centroid and application of the averaging method allowed one to find different peculiarities in the dynamics of hose-modulation instability too.

I. PROBLEM FORMULATION: BASIC EQUATIONS

Let us consider the trajectory of the wave-beam centroid, the spatial evolution of which, as it propagates along z, is governed by the Schrödinger equation

$$i\frac{\partial\Psi}{\partial z}+\Delta_\perp\Psi-n\Psi=0, \tag{1}$$

where n is the perturbation of the medium refraction index caused by the effect of the field. This trajectory is described by the well-known equation

$$\frac{\partial^2\vec{R}}{\partial z^2}=-\int|\Psi|^2\nabla_\perp n\,d\vec{r}\bigg/\int|\Psi|^2 d\vec{r},$$
$$\vec{R}=\int\vec{r}_\perp|\Psi|^2 d\vec{r}\bigg/\int|\Psi|^2 d\vec{r}. \tag{2}$$

Usually a special case of a medium with local nonlinearity is considered. When $n=F(|\Psi|)$, the centroid of the wave field Ψ moves along the straight line determined by the condition at the boundary of the nonlinear medium $(z=0)$. One cannot make such a univalent conclusion for the case of a medium with inertia of the nonlinear response when n depends on the field distribution at previous time moments.

Moreover, one can show a possible mechanism of the centroid motion instability. If in the field-induced perturbation of the refraction index an asymmetric mode can exist along with the initial symmetric mode (pump wave), then the right-hand side (force) of the motion equation (2) proves to be nonzero. Acceleration of the centroid motion, in turn, can lead to amplification of the nonsymmetric part of the field. Note that this instability mechanism is possible in a steady-state medium with nonlocal nonlinearity too.

Let us consider the dynamics of the wave-beam centroid behavior in a nonstationary medium. For this, the following expression describing relaxation of the nonlinear response is used:

$$\frac{\partial n}{\partial t}+n=\alpha|\Psi|^2. \tag{3}$$

This simplest case of Kerr-type nonlinearity relaxation has a number of advantages at the first stage of the construction of a self-consistent scenario. It is local relative to all spatial variables and in the steady-state case it corresponds to cubic nonlinearity. It allows the consideration of both nonlinearities: the focusing $(\alpha=-1)$ and defocusing $(\alpha=1)$ types.

Section IV describes also the centroid trajectory in a medium with a nonlinearity relaxation determined by the oscillator equation

$$\frac{\partial^2 n}{\partial t^2}+n=-|\Psi|^2. \tag{4}$$

Then the set of equations (1) and (4) describes the short laser pulse self-action in plasma due to a wake-field excitation.

II. GENERALIZATION OF PARAXIAL OPTICS APPROXIMATION

For a qualitative analysis of wave-beam variations it is convenient to use new variables z, t, and

$$\vec{\xi}=\vec{r}-\vec{R}(z,t). \tag{5}$$

As a result, the material equation (3) will be rewritten as

$$\frac{\partial n}{\partial t}-\vec{R}_t\cdot\frac{\partial n}{\partial \xi}+n=\alpha|\Psi|^2 \tag{6}$$

and the equation for the centroid motion will stay the same since only the integration variable in Eq. (2) was changed ($\vec{r}\to\vec{\xi}$). The Schrödinger equation in these variables,

$$i\left(\frac{\partial\Psi}{\partial z}-\vec{R}_z\cdot\frac{\partial\Psi}{\partial\vec{\xi}}\right)+\Delta\Psi-n\Psi=0, \tag{7}$$

by means of linear phase correction

$$\Psi=\Psi_H\exp(i\vec{R}_z\cdot\vec{\xi}), \tag{8}$$

is transformed as

$$i\left(\frac{\partial\Psi}{\partial z}+\vec{R}_z\cdot\frac{\partial\Psi}{\partial\vec{\xi}}\right)+\Delta\Psi-(n+\vec{R}_{zz}\cdot\vec{\xi})\Psi=0, \tag{9}$$

where the subscript H is omitted.

This equation describes the evolution of the wave beam as it propagates along a slightly curved trajectory. The effective potential $n+\vec{R}_{zz}\cdot\vec{\xi}$ is symmetric within the paraxial optics approximation, as is easily seen when using the equations for the trajectory (2). Having neglected higher-order aberrations, it is natural to assume that $|\Psi|^2$ stays an even function of ξ. The deviation of the trajectory from the straight line and the arising of instability are connected, within this approximation, with the asymmetric part of the refraction index perturbation. Therefore, we will represent n as a sum of symmetric S and asymmetric N parts $n=S+N$. Finally, within the approximation under consideration we obtain the following self-consistent system of equations, which is convenient for analysis:

$$\frac{\partial S}{\partial t}+S=\alpha|\Psi|^2+\vec{R}_t\cdot\frac{\partial N}{\partial\xi}, \tag{10}$$

$$\frac{\partial N}{\partial t}+N=\vec{R}_t\cdot\frac{\partial S}{\partial\vec{\xi}}, \tag{11}$$

$$\frac{\partial^2\vec{R}}{\partial z^2}=-\int\frac{\partial N}{\partial\vec{\xi}}|\Psi|^2d\vec{\xi}\bigg/\int|\Psi|^2d\vec{\xi}, \tag{12}$$

$$i\left(\frac{\partial\Psi}{\partial z}+\vec{R}_z\frac{\partial\Psi}{\partial\vec{\xi}}\right)+\Delta\Psi-S\Psi=0. \tag{13}$$

Hence one can see a rather strong effect of the symmetric part of the refraction index perturbation and weak reverse influence of the asymmetric part of the refraction index on the wave-beam self-action. This is seen with extraordinary clarity within the paraxial optics approximation. Having approximated the symmetric part of the refraction index with a parabola $S=S_0+S_2\xi^2/2$ and the asymmetric part with the expression $N=N_0\xi$ linear relative to ξ, it is easy to obtain, for Gaussian wave beam

$$\Psi=\frac{\sqrt{P}}{a}\exp\left(-\frac{\xi^2}{2a^2}\right)+i\Phi\xi^2, \tag{14}$$

the following equation that generalizes equations of the paraxial optics for the case of a non-rectilinear plane trajectory:

$$\frac{\partial^2 a}{\partial l^2}=\frac{1}{a^3}+S_2a,\quad\frac{\partial S_2}{\partial t}+S_2=\alpha\frac{P}{a^4}; \tag{15}$$

$$\frac{\partial^2 X}{\partial z^2}=N_0,\quad\frac{\partial N_0}{\partial t}+N_0=X_tS_2, \tag{16}$$

where within the approximation under cosideration we have, for length l,

$$\frac{\partial l}{\partial z}=1+\frac{1}{2}X_z^2. \tag{17}$$

The latter relationship demonstrates the weak reverse influence of the curvilinear trajectory on the spatial evolution of the wave beam propagating along the self-consistent trajectory. This is connected with the proper choice of the (z,ξ) coordinates. In the initial (rectangular) system of coordinates (z,v) the interaction is much stronger, even within the paraxial approximation.

To illustrate the effect of the centroid trajectory deviation from a straight line in an initially homogeneous medium let us consider the initial stage of self-effect process $(\partial n/\partial t\gg n)$. Having assumed that the structure of the wave beam is unperturbed ($a=a_0=$const), from Eq. (15) we find

$$S_2=\alpha\frac{P}{a_0^4}t. \tag{18}$$

The two consequent equations, within the approximation, yield

$$\frac{\partial^2 X_t'}{\partial z^2}=\alpha tX_t'\frac{P}{a_0^4}, \tag{19}$$

i.e., the effect of the trajectory deviation from the linear is determined by parameter $\beta=P/a^4$.

Note that Eq. (19) has the solution $X_t=0$, which corresponds to an invariable (stationary) trajectory. This is connected with the absence (neglect) of wave-field perturbations in the regime considered. A more accurate equation includes evidently the term proportional to $\partial|\Psi|^2/\partial t$ as a source.

Further, it is natural to separate the cases of the (i) focusing ($\alpha<0$) and (ii) defocusing ($\alpha>0$) nonlinearities.

(i) For the defocusing nonlinearity ($\alpha=1$) the equation for the trajectory leaving the coordinates origin is governed by

$$X=\int_0^t A(t)\sinh\sqrt{\beta t}z\,dt, \tag{20}$$

where $A(t)$ is an arbitrary function of time. In the simplest case $A\sim\alpha_0/\sqrt{\beta t}$, having integrated Eq. (20) we find

$$X=\frac{A_0}{\beta z}[\cosh(\sqrt{\beta t})z-1]. \tag{21}$$

It is evident that in the nonlinear regime ($\beta\gg1$) the trajectory deviates from the linear one exponentially:

$$X_L=A_0tz. \tag{22}$$

In the process of relaxation of the self-consistent distribution of the field and the medium refraction index the trajectory of the centroid straightens up and becomes the same as the one in the linear medium. The expressions for the symmetric and asymmetric parts of the perturbation of the medium refraction index derived from Eqs. (15) and (16) were used to describe the process of relaxation to steady-state trajectory

$$S_2\simeq\frac{P}{a^4},\quad N_0\simeq X_tS_2, \tag{23}$$

where $a(z)$ is determined by the stationary solution of Eqs. (14) and (18) and depends on the power P. As a result, for the trajectory in the plane (x,z) one can write the equation

$$\frac{\partial^2X}{\partial z^2}=\beta\frac{\partial X}{\partial t}, \tag{24}$$

i.e., the relaxation process is governed by the diffusion equation. Hence the time of stationary trajectory relaxation along the path of length L is

$$\tau\sim\frac{PL^2}{a^4}. \tag{25}$$

While estimating the wave-beam width along the path of its propagation within paraxial approximation

$$a^2=a_0+\frac{PL^2}{a_0^2}, \tag{26}$$

we finally obtain

$$\tau\simeq\frac{PL^2}{a_0^2+PL^2/a_0^2}, \tag{27}$$

where a_0 is the wave-beam width at $z=0$. One can see that the piece of the trajectory corresponding to the maximum relaxation time lies in the nonlinear focal region. It is easy to make the estimate that, with respect to its order of magnitude, this time is equal to the characteristic time of nonlinearity relaxation.

(ii) In the case of the focusing nonlinearity ($\alpha=-1$) there is no instability, as seen from Eq. (20), where the hyperbolic function is to be replaced by the trigonometric one. This conclusion is valid within the more accurate approach, which takes into account the fact that at times under consideration the regime is essentially nonlinear and the distribution formed is of the type of the homogeneous compressed filament [4] [$a\sim\exp(-Pt/4)$]. Similar calculations for such a dynamic structure show that the amplitude of the centroid oscillations decreases faster than the beam width.

The above study demonstrates that one can propose a mechanism for nonstable motion of the wave-field centroid, which is more adequate for this type of nonlinear response. In the case of defocusing nonlinearity the distribution of the refraction index is formed with a minimum at the system axis. The nonstable behavior of the central ray, to which the trajectory of the wave-beam centroid corresponds, is well known for this case. When the beam deviates, the latter goes away from the system axis. On the other hand, the formation of a perturbation of the refraction index with its maximum at the system axis in the case of focusing nonlinearity assists in stabilizing the behavior of the central ray.

III. NUMERICAL STUDY OF NONSTATIONARY SELF-ACTION

The long-term evolution of the wave beam was considered both from the basic equations and from the equations of the paraxial approximation taking into account the self-consistent variation of the central trajectory (15)–(17). The effect of a noticeable deviation from the straight line is determined by the parameter $\beta=P/A^4>1$. First let us represent the results of the numerical consideration of the paraxial approximation equations (15)–(17). In order to partially compensate the rather strong self-defocusing of the radiation (as the pattern becomes stationary at the conditions under consideration), the wave beams focusing at the rear boundary of the considered range were set.

The results of numerical consideration are shown in Fig. 1. The following parameters were chosen: $P=5$, $a=\sqrt{2}$, $a_z=0.2$, $X=0$, $X_z=0$ at $z=0$, $s_2=0$, and $N_0=0.03$ at $t=0$.

It is seen that the maximum deviation of the centroid trajectory from the straight line is reached at times $t\leqslant1$, until the width of the wave beam does not change noticeably. In the process of stabilization of the stationary field distribution the trajectory straightens up.

In the case of focusing nonlinearity ($\alpha=-1$) numerical calculations show that the motion of the centroid proves to have spatially periodic character. Its amplitude decreases rather fast in time. Thus the regime of collapse is a stable process relative to the aberration of the amplitude distribution and asymmetric bending of the phase front.

The paraxial optics approximation, as usual, describes only basic features of the changes in the wave-beam propa-

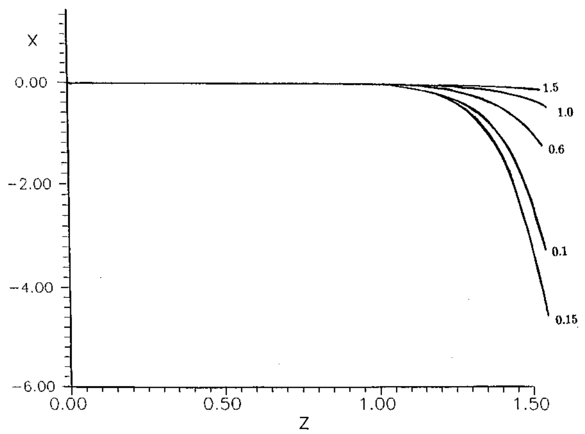

FIG. 1. Dynamics of the wave-field centroid trajectory in the case of defocusing nonlinearity within nonaberrational approximation, for $t=0.1,0.15,0.6,1.0,1.5$.

gation trajectory during its self-action in the media with inertia of the nonlinear response. The part of aberrations and, in particular, third-order aberrations in the case of nonstationary self-action considered may be significant, as seen from Eq. (9). The numerical investigation of the initial set of equations (1) and (3) was performed for the one-dimensional (in the transverse direction) Laplacian.

To retain the possibility of the wave collapse in the two-dimensional case we considered the higher-order nonlinearity. Namely, the following set of equations was considered:

$$i\frac{\partial\Psi}{\partial z}+\frac{\partial^2\Psi}{\partial x^2}-n\Psi=0, \tag{28}$$

$$\frac{\partial n}{\partial t}+n=\alpha|\Psi|^{2q} \tag{29}$$

at $q=2$, for which the collapse occurs in the case of focusing nonlinearity ($\alpha<0$).

Studies of the wave-field evolution were performed for a Gaussian shaped beam at the nonlinear medium boundary ($z=0$)

$$\Psi=\Psi_0\exp-\frac{x^2}{2(1+iz_F)}+i\phi x. \tag{30}$$

In the case of defocusing nonlinearity ($\alpha=1$) parameters were chosen according to the initial data of numerical calculations within the paraxial optics approximation. The beams were assumed to be focusing ($z_F=1$). In the case of focusing nonlinearity ($\alpha=-1$) the beams were assumed to be collimated ($z_F=0$), which allowed making the time of singularity formation somewhat longer. The coefficient of the linear aberration of the phase front was taken to be equal to $\phi=0.03$.

The results of computations show that aberrations change significantly the scenario of nonstationary self-action as compared to the paraxial optics approximation (Fig. 2). In the case of defocusing nonlinearity the stationary field distribution is finally realized, when the centroid of this field moves along a straight line. This fact was used to test numerical calculations. Aberrations lead to noticeably longer duration of the process of pattern stabilization. Several characteristic stages may be singled out in the process dynamics (see Figs. 2 and 3). First of all, it is seen that the time of the first maximum deviation of the wave-beam centroid from the initial trajectory (straight line) becomes an order longer. At this stage of nonstationary self-action the determinative part is played, as seen from Fig. 3, by inhomogeneities of the medium refraction index perturbation, which are excited near the front boundary of the nonlinear medium.

Having achieved the maximum deviation, the centroid trajectory begins to oscillate within the region determined by this curve and a final state. After several oscillations, the number of which grows with an increase of the field amplitude, a quasistationary trajectory is established.

At the last stage the trajectory straightens up rather slowly. This is a consequency, and, evidently, a fine indicator of the smoothening of the field distribution in the process of stabilization of the steady state.

Similar calculations for the case of focusing nonlinearity ($\alpha=-1$) were performed for a collimated wave beam ($z_F=0$). The amplitude Ψ_0 was chosen such as to provide a realization of the collapse dynamics at $\phi=0$ [4]. First of all, here it should be noted that the deviation of the centroid from the linear trajectory (Fig. 4) is much less than that of the defocusing nonlinearity under similar conditions. In accordance with the above qualitative analysis and numerical consideration within the paraxial optics approximation, the trajectory of the centroid motion proves to be an oscillating function along the trajectory of the wave beam. The oscillation period and amplitude decrease in time. The arising aberrations lead to a significant difference of the field structure from the initial Gaussian one. At times $t\simeq 1$ the asymmetry in the field distribution also is clearly observed (see Fig. 5). A comparison of the field growth rate for $\phi=0.03$ with the results for $\phi=0$ (symmetric condition) shows that the collapse is slower in the case under consideration. Unfortunately, this interesting effect of the collapse rate decreasing at weak aberrations of symmetry in the initial field distribution was studied only for times of the order of the times of nonlinear response relaxation. Then aberrations and fast spatial oscillations of the centroid lead to the destruction of the integrals in the initial system of equations and the reliability of the computations noticeably reduces.

IV. HOSE-MODULATION (SNAKELIKE) INSTABILITY

A different peculiar instability associated with excitation of a plasma wave by a long pulse was described in [2]. We show its mechanism based on the set of equations (1) and (4), taking no account of the plasma repulsion in the transverse direction due to pondermotive force. The corresponding equation for the motion of the wave-field centroid $\overline{x(z,\tau)}=\int x|A|^2dx\,dy$ has the form

$$\frac{\partial^2\overline{x}}{\partial z^2}=\int|A|^2\frac{\partial n}{\partial\xi}d\xi\,dy\Bigg/\int|A|^2dx\,dy, \tag{31}$$

where the coordinate $\xi=x-\overline{x}$ describes the shift of the wave beam relative to the straight line $x=0$. Equation (4) for the plasma wave potential in variables ξ and τ will be rewritten as

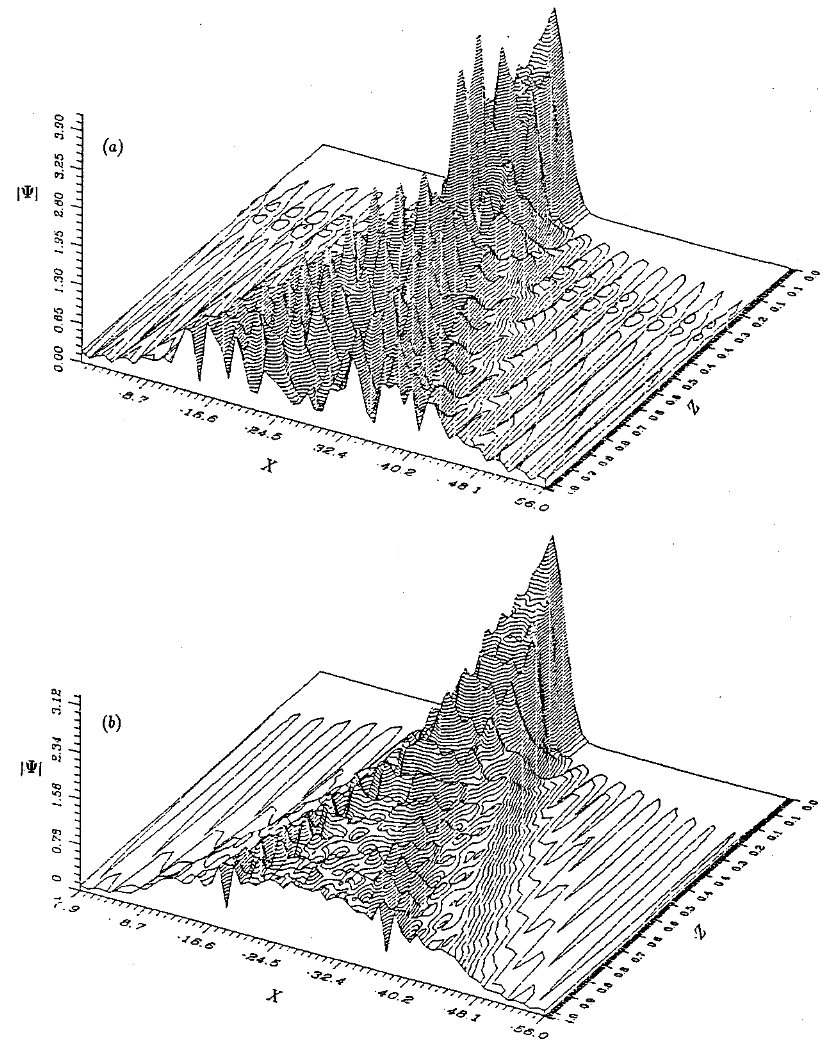

FIG. 2. Spatial distribution of the wave field in the case of defocusing nonlinearity for (a) $t=4$ and (b) $t=20$.

$$n_{\tau\tau}-2\overline{x_\tau}n_{\tau\xi}-\bar{x}_{\tau\tau}n_\xi+\overline{x_\tau^2}n_{\xi\xi}+n=|\Psi|^2. \tag{32}$$

Hence two possible ways for the instability to occur are evident. The first one is connected with the generation of an asymmetric mode by a symmetric wave beam at the displacement of the centroid. Such an asymmetric mode is an eigenmode either in the initially inhomogeneous distribution of the refraction index (e.g., in the plasma channel) or in the self-consistent process of the formation of a similar distribution as a result of the field effect.

Another possibility is associated with the excitation of the asymmetric part of the potential n in a Gaussian wave beam $|A|^2=(P/a^2)\exp[-(\xi^2+y^2)/a^2]$. Let us represent n as the sum of the symmetric (S) and asymmetric (N) parts, the equations for which, in the case of small perturbations, have the form

$$S_{\tau\tau}+S=|A|^2, \tag{33}$$

$$N_{\tau\tau}+N=2\overline{x_\tau}S_{\tau\tau}-\overline{x_{\tau\tau}}S_\xi. \tag{34}$$

Having integrated the equation for the symmetric part (33) for the case of a rectangular shape, $P(\tau)=A^2=\text{const}$, we find

$$S\simeq|A|^2(1-\cos\tau). \tag{35}$$

When deducing this expression, it was assumed that the width of the beam a actually stays the same along the plasma wavelength. As a result, the self-consistent set of equations for the centroid motion at the excitation of the asymmetric part of the potential takes on the form

$$N_{\tau\tau}+N=[2\overline{x_\tau}\sin\tau-(1-\cos\tau)\overline{x_{\tau\tau}}]|A|^2_\xi, \tag{36}$$

$$\frac{\partial^2\overline{x}}{\partial z^2}=\int|A|^2\frac{\partial N}{\partial\xi}d\xi\,dy\Big/\int|A|^2dx\,dy. \tag{37}$$

Now let us represent the solution of these equations as a sum of a slowly varying and quickly oscillating parts $\overline{x}=X+\tilde{x}\exp(i\tau)$ and $N=N+\tilde{N}\exp(i\tau)$. The amplitudes $\tilde{x}$ and $\tilde{N}$ are smooth on the scale of the function plasma period. Having separated the smooth and oscillating motions and averaging, in time we obtain the equations for slow amplitudes

$$\frac{\partial^2X}{\partial z^2}=-\tilde{x}\int(|A|^2_\xi)^2d\xi\,dy\Big/2\int|A|^2d\xi\,dy, \tag{38}$$

$$\frac{\partial^2\tilde{x}}{\partial z^2}=\int|A|^2\frac{\partial\tilde{N}}{\partial\xi}d\xi\,dy\Big/\int|A|^2d\xi\,dy, \tag{39}$$

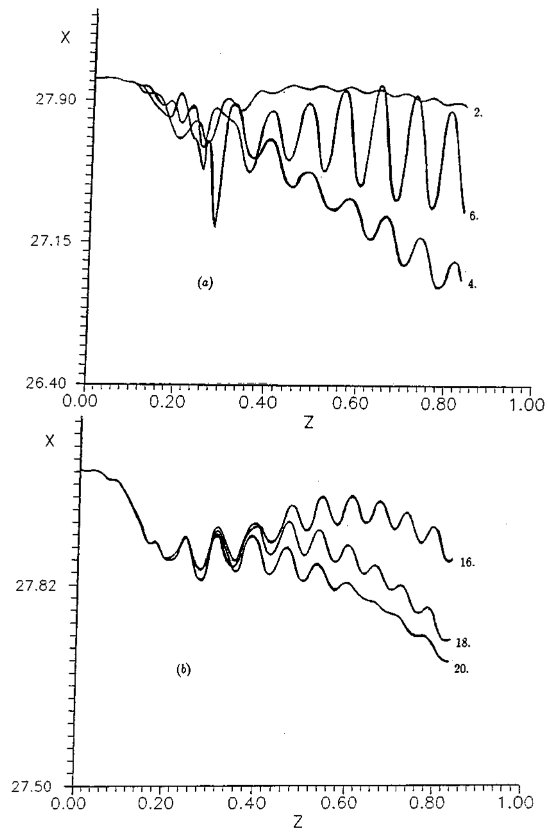

FIG. 3. Dynamics of the centroid trajectory under the conditions corresponding to Fig. 2 for (a) $t=2,4,6$ and (b) $t=16,18,20$.

$$2i\frac{\partial \widetilde{N}}{\partial \tau}=|A|^2_{\xi}\widetilde{x}. \tag{40}$$

These equations have to be appended with an equation for the variation of the beam width $a(z,\tau)$ (for example, within the aberrationless approximation). Specifically, one can use a self-similar-type structure [4,5].

Considering further equations at $a=a_0=\text{const}$, the final set of linear equations becomes

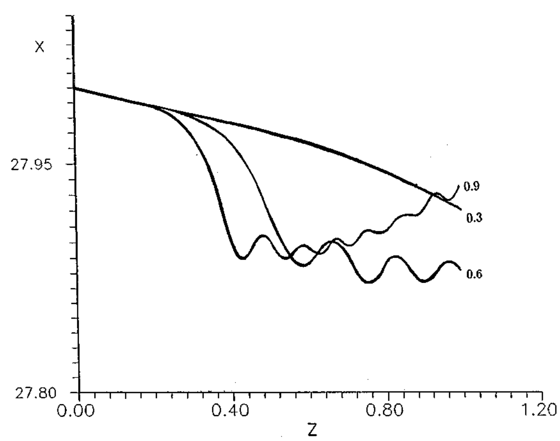

FIG. 4. Dynamics of the centroid trajectory in the case of focusing nonlinearity with initial conditions $\Psi_0=2.2$, $\Phi=0.03$, and $Z_F=0$ for $t=0.3,0.6,0.9$.

$$\frac{\partial^2 X}{\partial z^2}=-\frac{P\widetilde{x}}{4a^4}, \tag{41}$$

$$\frac{\partial^2 \widetilde{x}}{\partial z^2}=q, \tag{42}$$

$$i\frac{\partial \widetilde{q}}{\partial \tau}=-\frac{P\widetilde{x}}{4a^4}, \tag{43}$$

where $q=\int|A|^2(\partial\widetilde{N}/\partial\xi)d\xi\,dy/\int|A|^2 d\xi\,dy$. This yields a simple correlation between slow displacement of the centroid and the amplitude of the oscillating motion:

$$X=i\frac{\partial \widetilde{x}}{\partial \tau}. \tag{44}$$

Let us perform the Laplace transform over z under zero boundary conditions in Eqs. (42) and (43), integrate Eq. (43) over τ, and perform the inverse Laplace transform. As a result, we obtain the expression

$$\widetilde{x}=\frac{q_0}{2\pi i}\int_{-i\infty+\delta}^{i\infty+\delta}\frac{1}{p^2}\exp\left(i\frac{\tau P}{8p^2a^4}\right)+pz\,dp. \tag{45}$$

Having calculated this integral by the saddle-point method for $(\tau P/4a^4)z^2 \gg 1$, in the case corresponding to the growing solution of Eqs. (42) and (43) we obtain

$$\widetilde{X}=\frac{q_0(\sqrt{3}+i)}{\sqrt{6\pi}\left(\frac{P\tau}{4a^4}\right)^{1/2}}\exp[\tfrac{3}{4}(\sqrt{3}+i)]\left(\frac{\tau P}{4a^4}z^2\right)^{1/3}. \tag{46}$$

Having differentiated (44) over the fast growing exponent, we will find the expression for the slow part of the centroid:

$$X=\frac{q_0(i-\sqrt{3})z}{8\sqrt{6\pi}\tau\left(\frac{P\tau z^2}{4a^2}\right)^{1/6}}\exp[\tfrac{3}{4}(\sqrt{3}+i)]\left(\frac{P\tau z^2}{4a^4}\right)^{1/3}. \tag{47}$$

Thus, along with the growth of the centroid oscillation amplitude at the plasma frequency, there is a slow displacement of an averaged (over the plasma wavelength) position of the centroid away from the system axis ($r=0$). A detailed numerical and analytical study of variations in the amplitude oscillations during pulse self-modulation is described in [2]. The slow motion of the centroid, as seen from Eq. (42), must manifest itself along sufficiently long trajectories ($z\gg 1$); therefore, it can hardly be seen from the results of [2]. The period of nonlinear oscillations of the averaged centroid is $T\simeq(2\pi)^3a^4/Pz^2$ and by its order of magnitude it is equal to the reverse growth rate of the perturbation growth in the system under consideration.

This study shows that in the regime of the nonstationary self-effect of the wave beam its centroid trajectory may

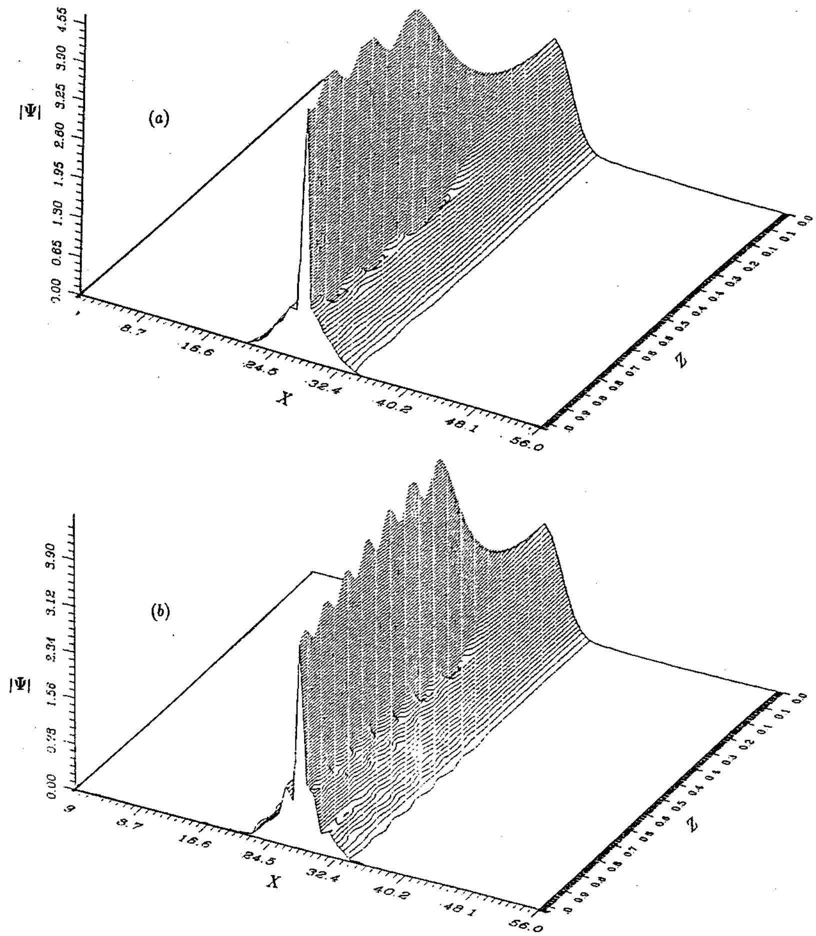

FIG. 5. Spatial distribution of the wave field in the case of focusing nonlinearity under the conditions corresponding to Fig. 4 for (a) $t=0.6$ and (b) $t=0.9$.

change significantly. The value of the wave-beam centroid deviation from the trajectory in a linear medium is determined by the parameter

$$P/a^4 \simeq \frac{E^2}{E_{cr}^2 a_0^2 k^2}, \tag{48}$$

where E_{cr} is the characteristic field of the nonlinearity type under consideration and k is wave number.

At $P/a^4 > 1$ the curvature (nonrectilinearity) of the wave-beam trajectory is to be taken into account. This parameter, as it is easily seen, contains the ratio of the power and critical self-focusing power in the numerator and the square of the Rayleigh length measured by radiation wavelengths in the denominator. It is obvious that the exceeding of a critical power determines the self-action dynamics (self-focusing, self-defocusing, self-modulation, etc.) while the symmetry of the problem remains undisturbed. In order to make apparent the considered effect of the wave-beam centroid deviation from a linear trajectory and stuctural changes due to the excitation of a nonsymmetric mode, the trajectories must exceed the Rayleigh length. The effect of disturbing the field distribution symmetry is realized more easily in the case of a hose instability developing [2]. In fact, for the pulse radiation one must replace P [see Eqs. (46) and (47)] by τP, where τ is the duration of electromagnetic radiation. Thus, for rather long pulses ($\tau \gg 1$) it is obvious that the dynamics of the centroid motion trajectory is determined by a weaker inequality ($P\tau/a^4 > 1$).

This effect manifests itself in different ways in media with defocusing and focusing nonlinearities. In the former case the bending of the phase front makes the process of stabilization of the stationary pattern longer and leads, in the nonstationary regime, to excitation of additional medium inhomogeneities with ''lifetimes'' of the order of the nonlinear relaxation time. In the medium with focusing nonlinearity, even in the symmetric case, the relaxation of the steady-state distribution proves to be a rather long process and runs in several stages (spatiotemporal collapse, structure instability and excitation of inhomogeneities, gradual displacement of inhomogeneities towards the rear boundary of the nonlinear medium, and only then formation of the stationary pattern). Deceleration of the initial (collapse) stage in this case may introduce significant changes to the relaxation process. It is evident that this will be assisted by structure instability, which leads to an extension of the region occupied by the field, as well as by the dynamics of inhomogeneities.

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